

Simulation of jet quenching and high- p_T particle production at RHIC and LHC

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The model to simulate rescattering and partonic energy loss in ultrarelativistic heavy ion collisions is presented. The full heavy ion event is obtained as a superposition of a soft hydro-type state and hard multi-jets. This model is capable of reproducing main features of the jet quenching pattern at RHIC, and is applied to probe jet quenching in various novel channels at LHC.

High- p_T physics at LHC

March 23-27 2007

University of Jyväskylä, Jyväskylä, Finland

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1. Introduction

One of the important tools for studying the properties of quark-gluon plasma (QGP) in ultrarelativistic heavy ion collisions is the analysis of a QCD jet production. The medium-induced energy loss of energetic partons, “jet quenching”, should be very different in the cold nuclear matter and QGP, resulting in many observable phenomena [1]. Recent RHIC data on high- p_T particle production at $\sqrt{s} = 200$ A GeV are in agreement with the jet quenching hypothesis (see, e.g., [2] and references therein). At LHC, a new regime of heavy ion physics will be reached at $\sqrt{s_{NN}} = 5.5$ A TeV where hard and semi-hard particle production can stand out against the underlying soft events. The initial gluon densities in PbPb reactions at LHC are expected to be much higher than those at RHIC, implying a stronger partonic energy loss, observable in new channels.

In the most of available Monte-Carlo heavy ion event generators the medium-induced partonic rescattering and energy loss are either ignored or implemented insufficiently. Thus, in order to analyze RHIC data on high- p_T hadron production and test the sensitivity of LHC observables to the QGP formation, the development of adequate and fast Monte-Carlo tool to simulate the jet quenching is necessary.

2. Physics model and simulation procedure

The detailed description of physics model can be found in our recent paper [3]. The approach bases on an accumulating energy loss, the gluon radiation being associated with each parton scattering in the expanding medium and includes the interference effect using the modified radiation spectrum dE/dl as a function of decreasing temperature T . The basic kinetic integral equation for the energy loss ΔE as a function of initial energy E and path length L has the form

$$\Delta E(L, E) = \int_0^L dl \frac{dP(l)}{dl} \lambda(l) \frac{dE(l, E)}{dl}, \quad \frac{dP(l)}{dl} = \frac{1}{\lambda(l)} \exp(-l/\lambda(l)), \quad (2.1)$$

where l is the current transverse coordinate of a parton, dP/dl is the scattering probability density, dE/dl is the energy loss per unit length, λ is in-medium mean free path. The collisional loss in high-momentum transfer limit and radiative loss in BDMS approximation [4] (with “dead-cone” generalization of the radiation spectrum for heavy quarks [5]) are using. We consider realistic nuclear geometry and treat the medium in nuclear overlapping zone as a boost-invariant longitudinally expanding quark-gluon fluid. The model parameters are the initial conditions for the QGP formation for central AuAu (PbPb) collisions at RHIC (LHC): the proper formation time τ_0 and the temperature T_0 . For non-central collisions we suggest the proportionality of the initial energy density ε_0 to the ratio of nuclear overlap function and transverse area of nuclear overlapping. The simple Gaussian parameterization of gluon angular distribution over the emission angle θ with the typical angle of the coherent radiation $\theta_0 \sim 5^\circ$ [6] is used.

The model was constructed as the Monte-Carlo event generator PYQUEN (PYthia QUENched) and is available via Internet [7]. The routine is implemented as a modification of the standard PYTHIA_6.4 jet event [8]. The event-by-event simulation procedure includes the generation of the

initial parton spectra with PYTHIA and production vertexes at given impact parameter, rescattering-by-rescattering simulation of the parton path length in a dense zone, radiative and collisional energy loss per rescattering, final hadronization with the Lund string model for hard partons and in-medium emitted gluons.

The full heavy ion event is simulated as a superposition of soft hydro-type state and hard multi-jets. The simple approximation [3] of hadronic liquid at “freeze-out” stage has been used to treat soft part of the event. Then the hard part of the event includes PYQUEN multi-jets generated according to the binomial distribution. The mean number of jets produced in AA events at a given impact parameter is a product of the number of binary NN sub-collisions and the integral cross section of hard process in pp collisions with the minimal transverse momentum transfer p_T^{\min} . The extended in such a way model has been also constructed as the fast Monte-Carlo event generator [9]. Note that ideologically similar approximation has been developed in [10].

3. Jet quenching at RHIC

In order to demonstrate the efficiency of the model, the jet quenching pattern in AuAu collisions at RHIC was considered. The PHOBOS data on η -spectra of charged hadrons [11] have been analyzed at first to fix the particle density in the mid-rapidity and the maximum longitudinal flow rapidity, $Y_L^{\max} = 3.5$ (figure 1). The rest of the model parameters were obtained by fitting PHENIX data on p_T -spectra of neutral pions [12] (figure 2): the kinetic freeze-out temperature $T_f = 100$ MeV, maximum transverse flow rapidity $Y_T^{\max} = 1.25$ and minimum transverse momentum of hard parton-parton scattering $p_T^{\min} = 2.8$ GeV/c. The nuclear modification of the hardest domain of p_T -spectrum was used to extract initial QGP conditions: $T_0 = 500$ MeV and $\tau_0 = 0.4$ fm/c. Figure 3 shows that our model well reproduces p_T - and centrality dependences of nuclear modification factor R_{AA} , which is defined as:

$$R_{AA}(p_T, \eta; b) = \frac{\sigma_{pp}^{\text{inel}}}{\langle N_{\text{coll}} \rangle} \frac{d^2 N_{AA}/dp_T d\eta}{d^2 \sigma_{pp}/dp_T d\eta},$$

where $\langle N_{\text{coll}} \rangle = T_{AA}(b) \times \sigma_{pp}^{\text{inel}}$ is the average number of binary nucleon-nucleon collisions in a given impact parameter b (with nucleus overlap function $T_{AA}(b)$). If there are no nuclear effects, the value of R_{AA} at high p_T should be unity.

Another important tool to verify jet quenching is two-particle azimuthal correlation function $C(\Delta\phi)$ – the distribution over an azimuthal angle of high- p_T hadrons in the event with $2 \text{ GeV}/c < p_T < p_T^{\text{trig}}$ relative to that for the hardest “trigger” particle with $p_T^{\text{trig}} > 4 \text{ GeV}/c$. Figure 4 presents $C(\Delta\phi)$ in pp and in central AuAu collisions (data from STAR [13]). Clear peaks in pp collisions at $\Delta\phi = 0$ and $\Delta\phi = \pi$ indicate a typical dijet event topology. However, for central AuAu collisions the peak near π disappears. It can be interpreted as the observation of monojet events due to the absorption of one of the jets in a dense medium. Figure 4 demonstrates that measured suppression of azimuthal back-to-back correlations is well reproduced by our model.

We leave beyond the scope of this paper the analysis of such important RHIC observables as the azimuthal anisotropy and particle ratios at low p_T . In order to study them, a more careful treatment of soft particle production than our simple approach is needed (the detailed description of space-time structure of freeze-out region, resonance decays, etc.).

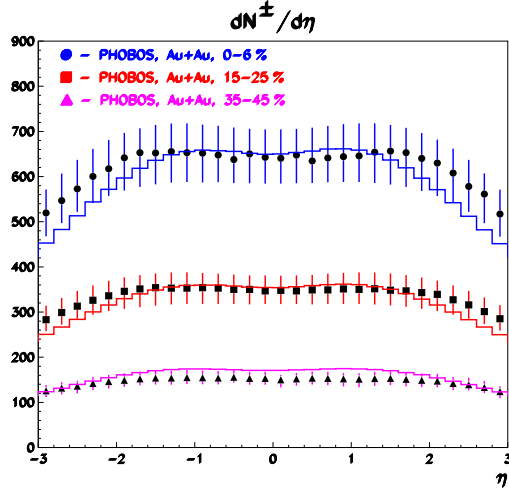


Figure 1: The pseudorapidity distribution of charged hadrons in AuAu collisions for three centrality sets. The points are PHOBOS data, histograms are the model calculations.

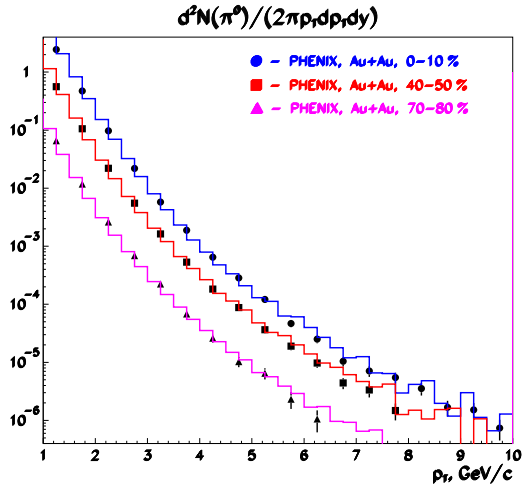


Figure 2: The transverse momentum distribution of neutral pions in AuAu collisions for three centrality sets. The points are PHENIX data, histograms are the model calculations.

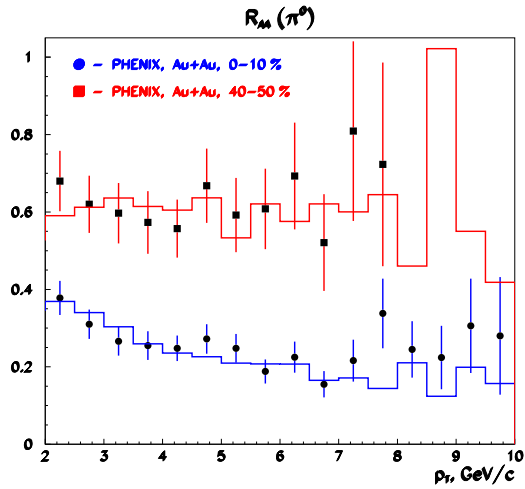


Figure 3: The nuclear modification factor R_{AA} for neutral pions in AuAu collisions for two centrality sets. The points are PHENIX data, histograms are the model calculations.

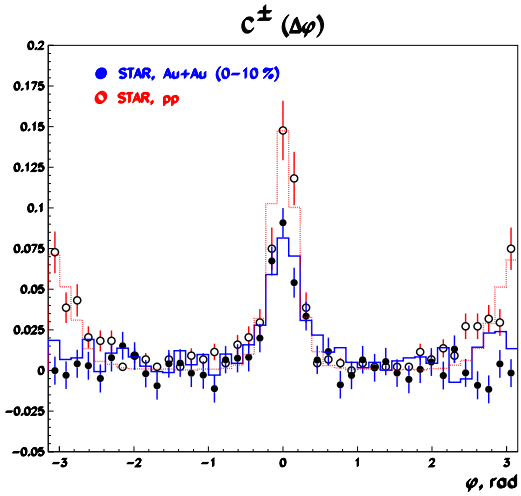


Figure 4: The azimuthal two-particle correlation function for pp and for central AuAu collisions. The points are STAR data, dashed and solid histograms are the model calculations for pp and AuAu events respectively.

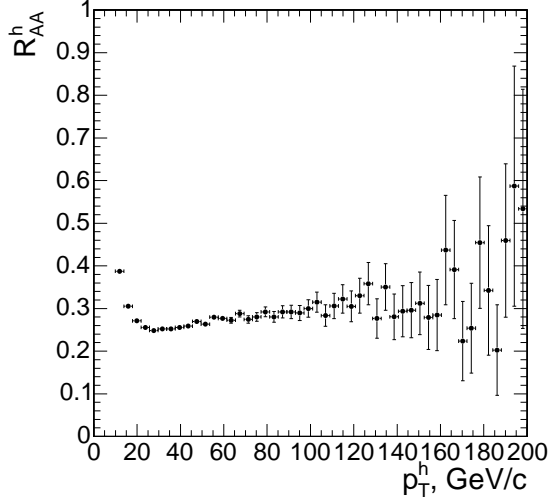


Figure 5: The nuclear modification factor, $R_{AA}^h(p_T)$, for inclusive charged hadrons in central PbPb collisions triggered on jets with $E_T^{\text{jet}} > 100$ GeV. The number of histogram entries and statistical errors correspond to the estimated event rate for one month of LHC run.

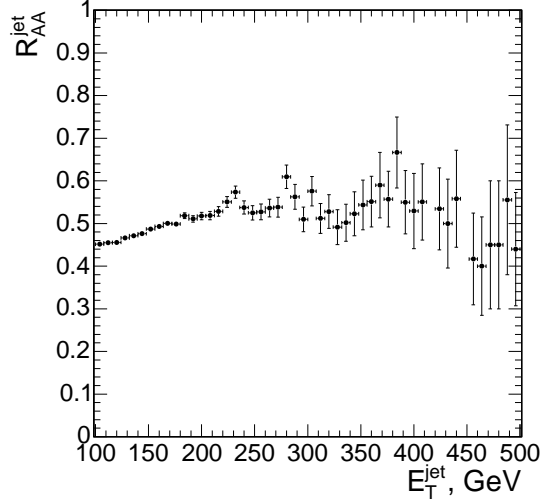


Figure 6: The nuclear modification factor, $R_{AA}^{\text{jet}}(p_T)$, for jets of cone size $R = 0.5$ in central PbPb collisions. The number of histogram entries and statistical errors correspond to the estimated event rate for one month of LHC run.

4. Jet quenching at LHC

The developed model was applied to analyze various novel features of jet quenching in heavy ion collisions at the LHC. Let us give a few examples of such jet observables. All calculations have been done for PbPb collisions at $\sqrt{s_{NN}} = 5.5$ TeV with PYQUEN energy loss model. The jet was defined on the generator level by a simple way, just collecting the energy around the direction of a leading particle inside a cone $R = \sqrt{\Delta\eta^2 + \Delta\phi^2} = 0.5$. The pseudorapidity cuts corresponding to the geometrical acceptance of CMS experiment were applied: $|\eta| < 3$ for jets and neutral hadrons, $|\eta| < 2.5$ for charged hadrons and muons.

4.1 Nuclear modification factors for jets

The nuclear modification factor can be determined for jets by the same way as for inclusive hadron production. Since at the LHC no pp data will be available at $\sqrt{s_{NN}} = 5.5$ TeV at the time of the first PbPb data taking, particle spectra in pp collisions will be interpolated to this energy using perturbative QCD predictions constrained by the existing Tevatron data at 1.8 TeV and by the LHC results at 14 TeV. Another possibility to quantifies medium-modified particle spectra is to use the central to peripheral heavy ion collision ratio, $R_{CP}(p_T, \eta)$, which does not require a pp reference, but has rather limited statistical reach of the peripheral data set.

Figure 5 shows the p_T -dependence of nuclear modification factor, $R_{AA}^h(p_T)$, for inclusive charged hadrons in central PbPb events triggered on jets with $E_T^{\text{jet}} > 100$ GeV. The number of entries and the statistical errors correspond to the estimated event rate for one month of LHC run and a nominal integrated luminosity of 0.5 nb^{-1} [14]. The estimated suppression factor slightly increases with $p_T (> 20 \text{ GeV})$, from ~ 0.25 at $p_T \sim 20 \text{ GeV}$ to ~ 0.35 at $p_T \sim 200 \text{ GeV}$. This

behaviour manifests the specific implementation of partonic energy loss in the model, rather weak energy dependence of loss and the shape of initial parton spectra. Without event triggering on high- E_T jet(s), the suppression is stronger (~ 0.15 at 20 GeV and slightly increasing with p_T up to ~ 0.3 at 200 GeV).

A novel observable at the LHC will be the nuclear modification factor for hard jets, which can be reconstructed in high multiplicity environment with a good efficiency and low background starting from the energy $E_T^{\text{jet}} \sim 50 - 100$ GeV [14]. Figure 6 shows the p_T -dependence of jet nuclear modification factor, $R_{\text{AA}}^{\text{jet}}(p_T)$. The other conditions are the same as it was described above. The estimated suppression factor (due to partial gluon bremsstrahlung out of jet cone and collisional loss) is about 2 and almost independent on jet energy. The measured jet nuclear modification factor will be very sensitive to the fraction of partonic energy loss carried out of the jet cone.

4.2 Medium-modified jet fragmentation function

The “jet fragmentation function” (JFF), $D(z)$, is defined as the probability for a given product of the jet fragmentation to carry a fraction z of the jet transverse energy. In nuclear (AA) interactions, the JFF for leading hadrons (i.e. the hadron carrying the largest fraction of the jet momentum) can be written as [15, 16]:

$$D(z) = \int_{z \cdot p_{T \min}^{\text{jet}}} d(p_T^h)^2 dy dz' \frac{dN_{\text{AA}}^{\text{h(k)}}}{d(p_T^h)^2 dy dz'} \delta\left(z - \frac{p_T^h}{p_T^{\text{jet}}}\right) \Bigg/ \int_{p_{T \min}^{\text{jet}}} d(p_T^{\text{jet}})^2 dy \frac{dN_{\text{AA}}^{\text{jet(k)}}}{d(p_T^{\text{jet}})^2 dy}, \quad (4.1)$$

where $p_T^h \equiv z p_T^{\text{jet}} = z' p_T$ is the transverse momentum of a leading hadron, z' is the hadron momentum fraction relative to the p_T of the parent parton, $p_{T \min}^{\text{jet}}$ is the minimum momentum threshold of observable jets, $(dN_{\text{AA}}^{\text{jet(k)}})/(d(p_T^{\text{jet}})^2 dy)$ and $(dN_{\text{AA}}^{\text{h(k)}})/(d(p_T^h)^2 dy dz')$ are the yields of k -type jets and hard hadrons, respectively.

Figure 7 shows JFF's in central PbPb collisions with and without partonic energy loss for $E_T^{\text{jet}} > 100$ GeV. The number of entries and the statistical errors again correspond to the estimated event rate for one month of LHC run. Significant softening of the JFF (by a factor of ~ 4 and slightly increasing with z) is predicted.

The medium-modified JFF is sensitive to a fraction ε of partonic energy loss carried out of the jet cone, which is related also to the suppression of the absolute jet rates. Figure 8 shows the ε -dependences of jet nuclear modification factor $R_{\text{AA}}^{\text{jet}}$ and ratio of JFF with energy loss to JFF without loss, $D^{\text{AA}}(z > z_0)/D^{\text{PP}}(z > z_0)$, for $z_0 = 0.5$ and 0.7 in central PbPb collisions [15, 16]. If ε close to 0, then $R_{\text{AA}}^{\text{jet}} \sim 1$ (there is no jet rate suppression), and JFF softening is maximal. Increasing ε results in stronger jet rate suppression, but effect on JFF softening becomes smaller, especially for highest z (the ratio $D^{\text{AA}}/D^{\text{PP}}$ can be even greater than 1 at large enough ε and z values). The physical reason for the effect to be opposite in the jet suppression factor and the fragmentation function is it follows. Increasing ε results in decreasing final jet transverse momentum (which is the denominator in definition of z in JFF (4.1)) without an influence on the numerator of z and, as a consequence, in reducing effect on JFF softening, while the integral jet suppression factor becomes larger. The crossing point between two effects is $\varepsilon \sim 0.3$.

Thus a novel concurrent study of the possible softening of the JFF and suppression of the absolute jet rates can be carried out in order to differentiate between various energy loss mechanisms.

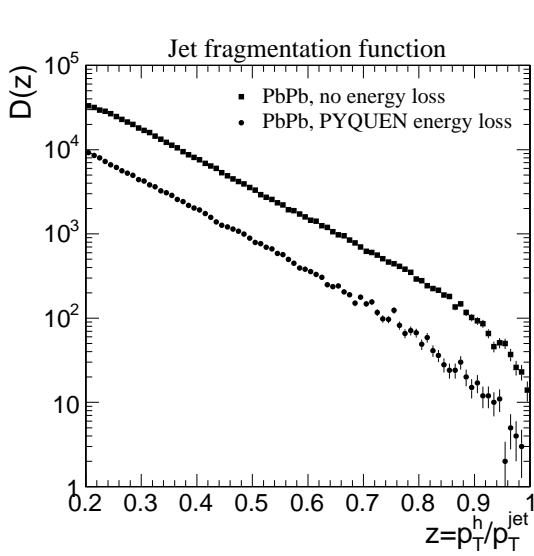


Figure 7: Jet fragmentation function for leading hadrons ($E_T^{\text{jet}} > 100$ GeV) in central PbPb collisions without (squares) and with (circles) partonic energy loss. The number of histogram entries and statistical errors correspond to the estimated jet rate for one month of LHC run.

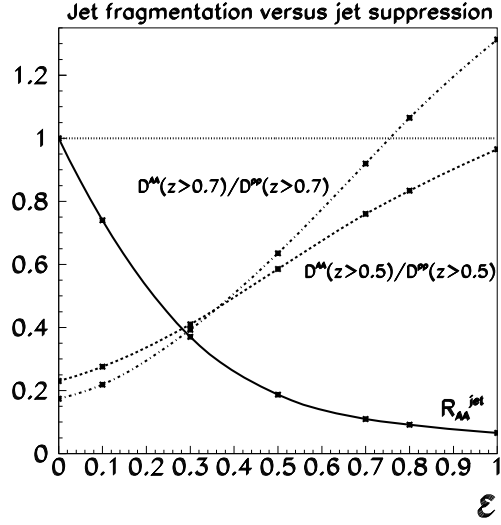


Figure 8: Jet nuclear modification factor (solid curve) and ratio of JFF with energy loss to JFF without loss ($z > 0.5$ for dashed curve and $z > 0.7$ for dash-dotted curve) as a function of the fraction ϵ of partonic energy loss carried out of the jet cone.

Strong JFF softening without substantial jet rate suppression would be an indication of small-angle gluon radiation dominating the medium-induced partonic energy loss. Increasing the contribution from wide-angle gluon radiation and collisional energy loss leads to jet rate suppression with less pronounced softening of the JFF. If, instead, the contribution of the “out-of-cone” jet energy loss is large enough, the jet rate suppression may be even more significant than the JFF softening.

4.3 Jet azimuthal anisotropy

The azimuthal anisotropy of particle spectrum is one of the most important tools to study properties of dense QCD-matter created in heavy ion collisions. It is usually characterized by the second coefficient of the Fourier expansion of particle azimuthal distribution, so called elliptic flow coefficient, v_2 . The momentum dependence of v_2 for high- p_T hadrons, observed in semi-central AuAu collisions at RHIC, strongly supports the presence of rescattering and energy loss of hard partons in the azimuthally asymmetric volume of the nuclear reaction. A novel observable at the LHC will be the azimuthal anisotropy for hard jets (due to the part of partonic energy loss carried out of jet cone).

The anisotropy of medium-induced partonic energy loss goes up with increasing collision impact parameter b , because the azimuthal asymmetry of the interaction volume gets stronger. However, the absolute value of the energy loss goes down with increasing b due to the reduced mean path length and the initial energy density. The non-uniform dependence of the loss on the parton azimuthal angle ϕ (with respect to the reaction plane) is then mapped onto the final parton spectra in semi-central collisions which are approximated well by the elliptic form [17, 18]. It results in the

elliptic anisotropy of observed high- p_T hadrons and hard jets. Figure 9 shows calculated impact parameter dependence of v_2 coefficient for jets with $E_T^{\text{jet}} > 100$ GeV and for inclusive charged hadrons with $p_T > 20$ GeV/ c in PbPb events triggered on jets. The absolute values of v_2 for high- p_T hadrons is larger than one's for jets by a factor of $\sim 2 - 3$. However, the shape of b -dependence of v_2^h and v_2^{jet} is similar: it increases almost linearly with the growth of the impact parameter b and becomes a maximum at $b \sim 1.6R_A$ (where R_A is the nucleus radius). After that, the v_2 coefficients drop rapidly with increasing b : this is the domain of impact parameter values, where the effect of decreasing energy loss due to a reducing effective transverse size of the dense zone and initial energy density of the medium is crucial and not compensated anymore by the stronger non-symmetry of the volume.

4.4 p_T -imbalance in dilepton tagged jet events

An important probe of medium-induced partonic energy loss in ultrarelativistic heavy ion collisions is production of a single jet opposite to a gauge boson such as a prompt γ [19] or a γ^*/Z^0 decaying into dileptons [20, 21]. The advantage of such processes is that the mean (i.e. averaged over all events) initial transverse momentum of the hard jet equal to the mean initial/final transverse momentum of boson, and the energy lost by the parton in the QCD medium can be directly estimated from the observed p_T -imbalance between the jet (or leading particle in a jet) and the lepton pair.

In the γ +jet case the main problem arises from the jet pair production background when a leading π^0 in the jet is misidentified as a photon. The “photon isolation” criteria usually used in pp collisions do not work with the same efficiency in high multiplicity heavy ion interactions. On the other hand, the production of jet tagged by dileptons is not affected significantly by backgrounds. The main background source – correlated semileptonic heavy quark decays – can be rejected using tracker information on the dilepton vertex position [14]. The moderate statistics, $\sim 500 - 1000$ $Z^0/\gamma^*(\rightarrow \mu^+\mu^-)$ +jet events per 1 month of LHC run with lead beams, are expected for the CMS geometrical acceptance and reasonable kinematic cuts [14, 22].

Figure 10 shows the difference between the transverse momentum of a $\mu^+\mu^-$ pair, $p_T^{\mu^+\mu^-}$, and five times the transverse energy of the leading particle in a jet (since the average fraction of the parent parton energy carried by a leading hadron at these energies is $z \approx 0.2$) for minimum bias PbPb collisions [22]. The process was simulated with CompHEP/PYTHIA generator package without and with partonic energy loss as obtained in the PYQUEN. The cuts $p_T^\mu > 5$ GeV/ c , $p_T^{\mu^+\mu^-} > 50$ GeV/ c and $E_T^{\text{jet}} > 50$ GeV, were applied. Despite the fact that the initial distribution is smeared and asymmetric due to initial-state gluon radiation, hadronization effects, etc., one can clearly see the additional smearing and the displaced mean and maximum values of the p_T -imbalance due to partonic energy loss. The p_T -imbalance between the $\mu^+\mu^-$ pair and a leading particle in a jet is directly related to the absolute value of partonic energy loss, and (unlike the p_T -imbalance between the $\mu^+\mu^-$ pair and jet itself) almost insensitive to the form of the angular spectrum of the emitted gluons or to the experimental jet energy resolution [22].

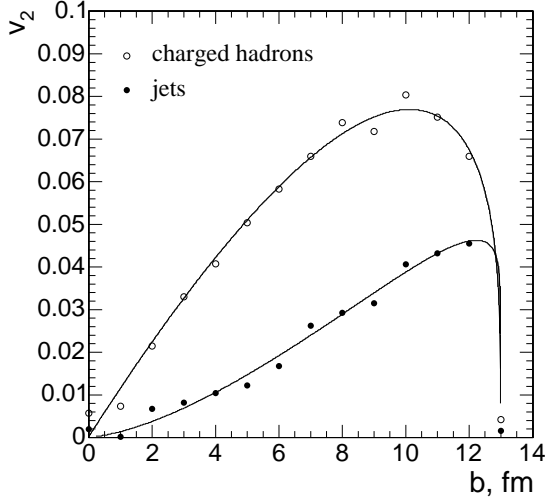


Figure 9: The impact parameter dependence of elliptic flow coefficients v_2^{jet} for jets with $E_T^{\text{jet}} > 100$ GeV (black circles) and v_2^h for inclusive charged hadrons with $p_T > 20$ GeV/c (open circles) in PbPb events triggered on jets.

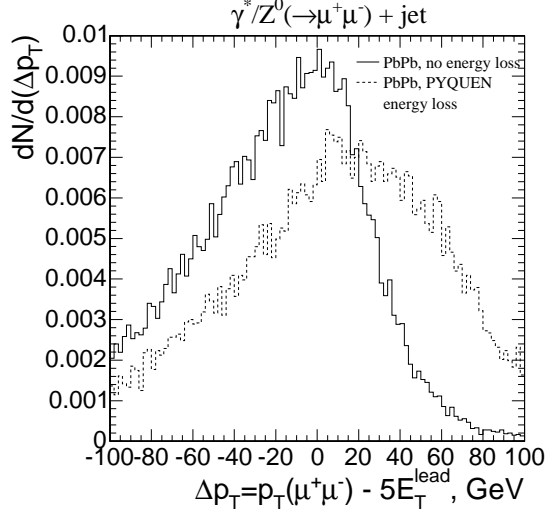


Figure 10: The distribution of the difference between the transverse momentum of a $Z^0/\gamma^* \rightarrow \mu^+\mu^-$ pair, $p_T^{\mu^+\mu^-}$, and five times the transverse energy of the leading particle in a jet, $5E_T^{\text{lead}}$, in minimum bias PbPb collisions with (dashed histogram) and without (solid histogram) partonic energy loss.

5. Conclusions

The method to simulate jet quenching in heavy ion collisions has been developed. The model is the fast Monte-Carlo tool implemented to modify a standard PYTHIA jet event. The full heavy ion event is obtained as a superposition of a soft hydro-type state and hard multi-jets. The model is capable of reproducing main features of the jet quenching pattern at RHIC: the p_T dependence of the nuclear modification factor and the suppression of azimuthal back-to-back correlations. The model has been applied to analyze new features of jet quenching pattern at LHC energy: jet nuclear modification factor, jet fragmentation function, jet azimuthal anisotropy and dilepton-jet correlations. The further development of the model focusing on a more detailed description of low- p_T particle production is in the progress.

Acknowledgments

Discussions with D. d’Enterria, I.M. Dremin, O.L. Kodolova, C. Loizides, A. Morsch, L.I. Malinina, C. Roland, L.I. Sarycheva, I.N. Vardanyan, G. Veres, I. Vitev, B. Wyslouch and B.G. Zakharov are gratefully acknowledged. I.L. thanks the organizers of the Workshop for the warm welcome and hospitality.

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